Finite Element Application in Reservoir Deformation Analysis (Pilot Phase 2)

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ABSTRACT

Finite element method (FEM) is a numerical technique for solving Engineering problem and Mathematical Physics. Useful problems with complicated geometries, Loading, and material properties where analytical solutions may not be obtained. Some of the complicated problems involving load is a cylindrical Reservoir structures where crude oil is stored in a Tank Farm. This paper demonstrates the use of Finite Element Analysis in above surface cylindrical Reservoir Engineering Structure. The reservoir which has sixteen (16) monitoring station was monitored using reflectorless Total station. This paper is the second phase of our pilot model and we hope to developed this second phase further to cover nine studs. The studs covered ranges from northern part of the Reservoir to the southern parts to further test the application of FEM triangular shaded (Truss) elements.

Keywords: Crude Oil, Finite Element, Reservoir, stiffness matrix, Strain and stress analysis.

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I. INTRODUCTION

The security of civil engineering structures demands a periodical monitoring. In many civil structures like bridges, vertical oil storage tanks, tunnels and dams; the deformations are the most critical parameters to be monitored. Monitoring the structural deformation and dynamic response to the large variety of external loadings has a great importance for maintaining structures safety and economical design of man-made structures. One of the Mathematical methods of deformation analysis is Finite Element method (FEM). Thus, the investigated deformable object must be treated as a mechanical system, which undergoes deformation according to the laws of continuum mechanics (Chrzanowski et al., 2006). This requires the causative factors (loads) of the process and the physical characteristics of the object under investigation to be included in both the design and analysis of the deformation. This is achieved by using deterministic modelling of the load-deformation relationship using finite element method (FEM) (Chrzanowski et al., 2006).

For simplicity, at this point, we assume a two-dimensional case with a single field variable $\phi(x, y)$ to be determined at every point $P(x, y)$ such that a known governing equation (or equations) is satisfied exactly at every such point. The basic idea in the finite element method is to find the solution of a complicated problem by replacing it by a simpler one. Since the actual problem is replaced by a simpler one in finding the solution, we will be able to find only an approximate solution rather than the exact solution (Yijun Liu, 2003). The existing mathematical tools will not be sufficient to find the exact solution (and sometimes, even an approximate solution) of

most of the practical problems. (Sourabh Vishwakarma et al 2017). Thus, in the absence of any other convenient method to find even the approximate solution of a given problem, we have to prefer the finite element method. Moreover, in the finite element method, it will often be possible to improve or refine the approximate solution by spending more computational effort (Singiresu, S. Rao, 2005).

II. LOCATION AND METHODS

The Forcados Yokri field is located in OMLs 43 and 45 in Burutu Local Government Area of Delta State of Nigeria. It is bounded approximately by the co-ordinates 319453mE to 335236mE and 148355mN to 141626mN (Ehigiator – Irughe 2019).

There are many severe consequences that could result from the failure of a large structure. In addition to jeopardizing public health and safety, environmental contamination and significant economic loss are also of major concern. It is for these reasons that any large deforming structure must be monitored to enable early detection of possible structural damage. Even a slight change of the object shape or changes to the surrounding area due to external factors (e.g., changes in ground water level or tectonic phenomena), no matter how insignificant they may appear, could compromise the integrity of a large structure and could lead to disaster. Most deformation monitoring schemes consist of measurements made to the monitored object that are referred to several reference points (assumed to be stable). To obtain correct object point displacements (and thus deformations), the stability of the reference points must be ensured. (Ehigiator –

Irughe 2019). The main conclusion from the many papers written on this topic states that every measurement made to a monitored object must be connected to stable control points. This is accomplished by creating a reference network of control points surrounding a particular structure (Ehigiator – Irughe et al, 2012).

Fig. 1. Reservoir no. 6.

III. MATERIALS AND METHODS

An effective approach is carried out to model the structure of oil storage tank by using well-chosen discrete monitoring points located on the surface of the structure at different levels which, when situated correctly, accurately depict the characteristics of the structure.

Any movements of the monitoring point locations (and thus deformations of the structure) can be detected by maintaining the same point locations over time and by performing measurements to them at specified time intervals enabling direct point displacement comparisons (Ehigiator – Irughe et al 2011). A common approach for this method is to place physical targets on each chosen discrete point to which measurements can be made. However, there are certain situations in which monitoring the deformations of a large structure using direct displacement measurements of targeted points is uneconomical, unsafe, inefficient, or simply impossible (Ehigiator – Irughe 2019). Reasons for this limitation vary, but it may be as simple as placement of permanent target prisms on the structure is too difficult or costly figure 2A below represent truss elements around the reservoir under study, while figure 2B represent the section of the Reservoir whose deformation status is been considered in this study ie studs 1 to 9.

a)

b)

Fig. 2. Reservoir cross section.

A. Determination of Stiffness Matrix for the Elements

The primary characteristics of a finite element are embodied in the element stiffness matrix. For a structural finite element, the stiffness matrix contains the geometric and material behavior information that indicates the resistance of the element to deformation when subjected to loading. Such deformation may include axial, bending, shear, and torsional effects (Ehigiator - Irughe, 2019). For finite elements used in nonstructural analyses, such as fluid flow and heat transfer, the term stiffness matrix is also used, since the matrix represents the resistance of the element to change when subjected to external influences. The equation for the local stiffness matrix is given by equation (1) below.

$$\left\{ k = \frac{AE}{l} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} \right\} \quad (1)$$

The Strain displacement Matrix [B] and the directional cosines l and m is given as in equation (2).

$$\left\{ [B] = \frac{1}{2A} \begin{bmatrix} \beta_1 & 0 & \beta_2 & 0 & \beta_3 & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & \beta_1 & \gamma_2 & \beta_2 & \gamma_3 & \beta_3 \end{bmatrix} \right\} \quad (2)$$

$$\{ l_1 = \cos \theta_1, m_1 = \sin \theta_1 \}$$

where $\beta_1 = y_2 - y_3, \beta_2 = y_3 - y_1, \beta_3 = y_1 - y_2;$

$\gamma_1 = x_2 - x_3, \gamma_2 = x_3 - x_1, \gamma_3 = x_1 - x_2;$

or

$\beta_1 = y_{23}, \beta_2 = y_{31}, \beta_3 = y_{12};$

$\gamma_1 = x_{23}, \gamma_2 = x_{31}, \gamma_3 = x_{12}.$

Stud 1 and stud2 represent node 1 and node 2 while center is node 0. Node 0 is common origin for other Truss element in this paper. Below are the measured coordinates of the nodal points. The nodal coordinates of elements 1 to 9 i.e. from figures 3A to 3H and the solutions of their local matrices are presented below.

Fig. 3A. Truss Element 1 for node 0, 1 and 2.

$$\begin{cases} Y_1 = 845.627m, X_1 = 619.627m \\ Y_2 = 859.699m, X_2 = 613.794m \\ Y_0 = 840.028m, X_0 = 581.201m \end{cases}$$

$$\tan \theta_1 = \frac{845.627 - 840.028}{619.005 - 581.201} = \frac{5.599}{37.804} = 0.14811$$

$$\theta_1 = 8^\circ 25' 29''$$

$$\theta_2 = \frac{859.699 - 845.622}{613.794 - 619.005} = \frac{14.077}{-5.211}$$

$$\theta_2 = -69^\circ 41' 12'' + 90 = 20^\circ 18' 48''$$

$$\theta_3 = \frac{840.028 - 859.699}{581.201 + 613.794} = \frac{-19.671}{-32.592} = 31^\circ 06' 45''$$

For Element 1

$$L_1 = \cos \theta_1 = \cos(8^\circ 25' 29'') = 0.989$$

$$M_1 = \sin \theta_1 = (8^\circ 25' 29'') = 0.147$$

$$A_1 = 280.577m^2$$

$$K_1 = \frac{280.577 \times 210 \times 10^9}{38.186} \begin{bmatrix} su_0 & sv_0 & su_1 & sv_1 \\ 0.978 & 0.145 & -0.978 & -0.145 \\ 0.145 & 0.022 & -0.145 & -0.022 \\ -0.978 & -0.145 & 0.975 & 0.145 \\ -0.145 & -0.022 & 0.145 & 0.022 \end{bmatrix} \begin{matrix} su_0 \\ sv_0 \\ su_1 \\ sv_1 \end{matrix}$$

$$K_2 = \frac{280.577 \times 210 \times 10^9}{15.006} \begin{bmatrix} su_1 & sv_1 & su_2 & sv_2 \\ 0.880 & 0.355 & -0.880 & -0.325 \\ 0.325 & 0.120 & -0.325 & -0.120 \\ -0.880 & -0.325 & 0.880 & 0.325 \\ -0.325 & -0.120 & 0.325 & 0.120 \end{bmatrix} \begin{matrix} su_1 \\ sv_1 \\ su_2 \\ sv_2 \end{matrix}$$

$$L_2 = \cos 20^\circ 18' 48''$$

$$L_2 = 0.938$$

$$M_2 = \sin 20^\circ 18' 48''$$

$$M_2 = 0.347$$

$$K_3 = \frac{280.577 \times 210 \times 10^9}{38.186} \begin{bmatrix} su_2 & sv_2 & su_0 & sv_0 \\ 0.733 & 0.443 & -0.733 & -0.443 \\ 0.443 & 0.267 & -0.443 & -0.267 \\ -0.733 & -0.443 & 0.733 & 0.443 \\ -0.443 & 0.267 & 0.443 & 0.267 \end{bmatrix} \begin{matrix} su_2 \\ sv_2 \\ su_0 \\ sv_0 \end{matrix}$$

Fig. 3B. Truss Element 2 for node 0, 2 and 3.

$$\begin{cases} Y_3 = 871.061m, X_3 = 603.075m \\ Y_2 = 859.699m, X_2 = 613.794m \\ Y_0 = 840.028m, X_0 = 581.201m \end{cases}$$

$$A_2 = 290.855$$

$$K_4 = \frac{290.855 \times 210 \times 10^9}{38.186}$$

$$\theta_4 = \frac{871.061 - 859.699}{603.075 - 581.201} = \frac{11.362}{21.874}$$

$$\theta_4 = 27^\circ 26' 55''$$

$$\theta_5 = \frac{840.028 - 871.061}{581.201 - 603.075} = \frac{-31.033}{-21.874}$$

$$\theta_5 = 54^\circ 49' 17''$$

$$K_4 = \frac{290.855 \times 210 \times 10^9}{15.803} \begin{bmatrix} su_2 & sv_2 & su_3 & sv_3 \\ 0.787 & 0.409 & -0.787 & -0.409 \\ 0.409 & 0.213 & -0.409 & -0.213 \\ -0.787 & -0.409 & 0.787 & 0.409 \\ -0.409 & -0.213 & 0.409 & 0.213 \end{bmatrix} \begin{matrix} su_2 \\ sv_2 \\ su_3 \\ sv_3 \end{matrix}$$

$$L_4 = \cos 27^\circ 26' 55'' = 0.887$$

$$M_4 = \sin 27^\circ 26' 55'' = 0.461$$

$$K_5 = \frac{290.855 \times 210 \times 10^9}{38.186} \begin{bmatrix} su_3 & sv_3 & su_0 & sv_0 \\ 0.332 & 0.471 & -0.332 & -0.471 \\ 0.471 & 0.667 & -0.471 & -0.667 \\ -0.332 & -0.471 & 0.332 & 0.471 \\ -0.471 & -0.667 & 0.471 & 0.667 \end{bmatrix} \begin{matrix} su_3 \\ sv_3 \\ su_0 \\ sv_0 \end{matrix}$$

$$L_5 = \cos 54^\circ 49' 17'' = 0.576$$

$$M_5 = \sin 54^\circ 49' 17'' = 0.817$$

Fig. 3C. Truss Element 3 for node 0, 3 and 4.

$$\begin{cases} Y_4 = 86.996m, X_4 = 590.075m \\ Y_3 = 871.061m, X_3 = 603.075m \\ Y_0 = 840.028m, X_0 = 581.201m \end{cases}$$

$$\theta_6 = \frac{876.996 - 871.661}{590.075 - 603.075} = \frac{5.935}{-13.00} = -24^\circ 32' 19'' + 90$$

$$\theta_6 = 65^\circ 27' 41''$$

$$\theta_7 = \frac{840.028 - 876.996}{581.201 - 590.075} = \frac{-36.968}{-8.874} = 76^\circ 30' 06''$$

$$K_6 = \frac{266.398 \times 210 \times 10^9}{14.276} \begin{bmatrix} su_3 & sv_3 & su_4 & sv_4 \\ 0.172 & 0.378 & -0.172 & -0.378 \\ 0.378 & 0.828 & -0.378 & -0.828 \\ -0.172 & -0.378 & 0.172 & 0.378 \\ -0.378 & -0.828 & 0.378 & 0.828 \end{bmatrix} \begin{matrix} su_3 \\ sv_3 \\ su_4 \\ sv_4 \end{matrix}$$

$$L_6 = \cos 65^\circ 27' 41'' = 0.415$$

$$M_6 = \sin 65^\circ 27' 41'' = 0.910$$

$$K_7 = \frac{266.398 \times 210 \times 10^9}{38.186} \begin{bmatrix} su_4 & sv_4 & su_0 & sv_0 \\ 0.054 & 0.226 & -0.054 & -0.226 \\ 0.226 & 0.945 & -0.226 & -0.945 \\ -0.054 & -0.226 & 0.054 & 0.226 \\ -0.226 & -0.945 & 0.226 & 0.945 \end{bmatrix} \begin{matrix} su_4 \\ sv_4 \\ su_0 \\ sv_0 \end{matrix}$$

$$L_7 = \cos 76^\circ 30' 06'' = 0.233$$

$$M_7 = \sin 76^\circ 30' 06'' = 0.972$$

Fig. 3D. Truss Element 4 for node 0, 4 and 5.

$$\begin{cases} Y_5 = 877.544m, X_5 = 575.137m \\ Y_4 = 876.996m, X_4 = 590.075m \\ Y_0 = 840.028m, X_0 = 581.201m \end{cases}$$

$$\theta_8 = \frac{877.544 - 876.996}{575.137 - 590.075} = \frac{0.548}{-14.938} = -2^\circ 06' 03'' + 90$$

$$\theta_8 = 87^\circ 53' 57''$$

$$\theta_9 = \frac{840.028 - 877.544}{581.201 - 575.137} = \frac{-37.516}{6.064} = -80^\circ 49' 06'' + 90$$

$$\theta_9 = 9^\circ 10' 54''$$

$$L_8 = \cos 87^\circ 53' 57'' = 0.037$$

$$M_8 = \sin 87^\circ 53' 57'' = 0.999$$

$$K_8 = \frac{279.298 \times 210 \times 10^9}{14.940} \begin{bmatrix} su_4 & sv_4 & su_5 & sv_5 \\ 1.369 \times 10^{-3} & 0.037 & -1.369 \times 10^{-3} & -0.037 \\ 0.037 & 0.998 & -0.037 & -0.998 \\ -1.369 \times 10^{-3} & -0.037 & 1.369 \times 10^{-3} & 0.037 \\ -0.037 & -0.998 & 0.037 & 0.998 \end{bmatrix} \begin{matrix} su_4 \\ sv_4 \\ su_5 \\ sv_5 \end{matrix}$$

$$L_9 = \cos 9^\circ 10' 54'' = 0.987$$

$$M_9 = \sin 9^\circ 10' 54'' = 0.160$$

$$K_9 = \frac{279.298 \times 210 \times 10^9}{38.186} \begin{bmatrix} su_5 & sv_5 & su_0 & sv_0 \\ 0.974 & 0.158 & -0.974 & -0.154 \\ 0.154 & 0.026 & -0.154 & -0.026 \\ -0.974 & -0.154 & 0.974 & 0.154 \\ -0.154 & -0.026 & 0.154 & 0.026 \end{bmatrix} \begin{matrix} su_5 \\ sv_5 \\ su_0 \\ sv_0 \end{matrix}$$

Fig. 3E. Truss Element 5 for node 0, 5 and 6.

$$\begin{cases} Y_6 = 872.371m, X_6 = 561.371m \\ Y_5 = 877.544m, X_5 = 576.137m \\ Y_0 = 840.028m, X_0 = 581.201m \end{cases}$$

$$\theta_{10} = \frac{872.371 - 877.5411}{561.096 - 576.137} = \frac{-5.173}{-15.041} = 18^\circ 58' 46''$$

$$\theta_{11} = \frac{840.028 - 872.371}{581.201 - 561.096} = \frac{-32.343}{20.105} = -58^\circ 08' 03'' + 90$$

$$\theta_{11} = 31^\circ 51' 57''$$

$$K_{10} = \frac{280.118 \times 210 \times 10^9}{14.958} \begin{bmatrix} su_5 & sv_5 & su_6 & sv_6 \\ 0.895 & 0.307 & -0.895 & -0.307 \\ 0.307 & 0.106 & -0.307 & -0.106 \\ -0.895 & -0.307 & 0.895 & 0.307 \\ -0.307 & -0.106 & 0.307 & 0.106 \end{bmatrix} \begin{bmatrix} su_5 \\ sv_5 \\ su_6 \\ sv_6 \end{bmatrix}$$

$$L_{10} = \cos 18^\circ 58' 46'' = 0.946$$

$$M_{10} = \sin 18^\circ 58' 46'' = 0.325$$

$$K_{11} = \frac{280.118 \times 210 \times 10^9}{38.186} \begin{bmatrix} su_6 & sv_6 & su_0 & sv_0 \\ 0.721 & 0.448 & -0.721 & -0.448 \\ 0.448 & 0.279 & -0.448 & -0.279 \\ -0.721 & -0.448 & 0.721 & 0.448 \\ -0.448 & -0.279 & 0.448 & 0.279 \end{bmatrix} \begin{bmatrix} su_6 \\ sv_6 \\ su_0 \\ sv_0 \end{bmatrix}$$

$$L_{11} = \cos 31^\circ 51' 57'' = 0.849$$

$$M_{11} = \sin 31^\circ 51' 57'' = 0.528$$

Fig. 3F. Truss Element 6 for node 0, 6 and 7.

$$\begin{cases} Y_7 = 862.193m, X_7 = 550.160m \\ Y_6 = 872.371m, X_6 = 561.098m \\ Y_0 = 840.028m, X_0 = 581.201m \end{cases}$$

$$\theta_{12} = \frac{862.193 - 872.372}{550.160 - 561.098} = \frac{-10.179}{-10.938} = 42^\circ 56' 30''$$

$$\theta_{13} = \frac{840.028 - 862.193}{581.201 - 550.160} = \frac{-22.165}{31.041} = -35^\circ 31' 44'' + 90$$

$$\theta_{13} = 54^\circ 28' 16''$$

$$L_{12} = \cos 42^\circ 56' 30'' = 0.732$$

$$M_{12} = \sin 42^\circ 56' 30'' = 0.681$$

$$L_{13} = \cos 54^\circ 28' 16'' = 0.581$$

$$M_{13} = \sin 54^\circ 28' 16'' = 0.814$$

$$K_{12} = \frac{278.043 \times 210 \times 10^9}{14.941} \begin{bmatrix} su_6 & sv_6 & su_7 & sv_7 \\ 0.536 & 0.249 & -0.536 & -0.249 \\ 0.249 & 0.464 & -0.249 & -0.464 \\ -0.536 & -0.249 & 0.536 & 0.249 \\ -0.249 & -0.464 & 0.249 & 0.464 \end{bmatrix} \begin{bmatrix} su_6 \\ sv_6 \\ su_7 \\ sv_7 \end{bmatrix}$$

$$K_{13} = \frac{278.043 \times 210 \times 10^9}{38.186} \begin{bmatrix} su_7 & sv_7 & su_0 & sv_0 \\ 0.338 & 0.473 & -0.338 & -0.473 \\ 0.473 & 0.663 & -0.473 & -0.663 \\ -0.338 & -0.473 & 0.338 & 0.473 \\ -0.473 & -0.663 & 0.473 & 0.663 \end{bmatrix} \begin{bmatrix} su_7 \\ sv_7 \\ su_0 \\ sv_0 \end{bmatrix}$$

Fig. 3G. Truss Element 7 for node 0, 7 and 8.

$$\begin{cases} Y_8 = 848.526m, X_8 = 543.963m \\ Y_7 = 862.193m, X_7 = 550.160m \\ Y_0 = 840.028m, X_0 = 581.201m \end{cases}$$

$$\theta_{14} = \frac{848.526 - 862.193}{543.963 - 550.160} = \frac{-13.667}{-6.197} = 65^\circ 36' 33''$$

$$\theta_{15} = \frac{840.028 - 848.526}{581.201 - 543.963} = \frac{-8.498}{37.238} = -12^\circ 51' 19'' + 90$$

$$\theta_{15} = 77^\circ 08' 41''$$

$$L_{14} = \cos 65^\circ 36' 33'' = 0.413$$

$$M_{14} = \sin 65^\circ 36' 33'' = 0.911$$

$$L_{15} = \cos 77^\circ 08' 41'' = 0.222$$

$$M_{15} = \sin 77^\circ 08' 41'' = 0.975$$

$$K_{14} = \frac{276.175 \times 210 \times 10^9}{15.004} \begin{bmatrix} su_7 & sv_7 & su_8 & sv_8 \\ 0.171 & 0.376 & -0.171 & -0.376 \\ 0.376 & 0.830 & -0.376 & -0.830 \\ -0.171 & -0.376 & 0.171 & 0.376 \\ -0.376 & -0.830 & 0.376 & 0.830 \end{bmatrix} \begin{bmatrix} su_7 \\ sv_7 \\ su_8 \\ sv_8 \end{bmatrix}$$

$$K_{15} = \frac{276.175 \times 210 \times 10^9}{38.186} \begin{bmatrix} su_7 & sv_7 & su_8 & sv_8 \\ 0.049 & 0.216 & -0.049 & -0.216 \\ 0.216 & 0.951 & -0.216 & -0.951 \\ -0.049 & -0.216 & 0.049 & 0.216 \\ -0.216 & -0.951 & 0.216 & 0.951 \end{bmatrix} \begin{bmatrix} su_7 \\ sv_7 \\ su_8 \\ sv_8 \end{bmatrix}$$

Fig. 3H. Truss Element 8 for node 0, 8 and 9.

$$\begin{cases} Y_9 = 834.428m, X_9 = 543.397m \\ Y_8 = 848.526m, X_8 = 543.963m \\ Y_0 = 840.028m, X_0 = 581.201m \end{cases}$$

$$\theta_{16} = \frac{834.428 - 848.526}{543.397 - 543.963} = \frac{-14.096}{0.566} = 87^\circ 42' 03''$$

$$\theta_{17} = \frac{840.028 - 834.428}{581.201 - 543.392} = \frac{5.600}{37.504} = 8^\circ 25' 34''$$

$$L_{16} = \cos 87^\circ 42' 03'' = 0.040$$

$$M_{16} = \sin 87^\circ 42' 03'' = 0.999$$

$$L_{17} = \cos 8^\circ 25' 34'' = 0.989$$

$$M_{17} = \sin 8^\circ 25' 34'' = 0.147$$

$$K_{16} = \frac{262.662 \times 210 \times 10^9}{14.110} \begin{bmatrix} su_8 & sv_8 & su_9 & sv_9 \\ 1.6 \times 10^{-3} & 0.040 & -1.6 \times 10^{-3} & -0.040 \\ 0.040 & 0.998 & -0.040 & -0.998 \\ -1.6 \times 10^{-3} & -0.040 & 1.6 \times 10^{-3} & 0.040 \\ -0.040 & -0.998 & 0.040 & 0.998 \end{bmatrix} \begin{matrix} su_8 \\ sv_8 \\ su_9 \\ sv_9 \end{matrix}$$

$$K_{16} = 3909 \times 10^9 \begin{bmatrix} su_8 & sv_8 & su_9 & sv_9 \\ 1.6 \times 10^{-3} & 0.040 & -1.6 \times 10^{-3} & -0.040 \\ 0.040 & 0.998 & -0.040 & -0.998 \\ -1.6 \times 10^{-3} & -0.040 & 1.6 \times 10^{-3} & 0.040 \\ -0.040 & -0.998 & 0.040 & 0.998 \end{bmatrix} \begin{matrix} su_8 \\ sv_8 \\ su_9 \\ sv_9 \end{matrix}$$

$$K_{17} = \frac{262.662 \times 210 \times 10^9}{38.186} \begin{bmatrix} su_9 & sv_9 & su_0 & sv_0 \\ 0.978 & 0.145 & -0.978 & -0.148 \\ 0.145 & 0.022 & -0.145 & -0.022 \\ -0.978 & -0.145 & 0.978 & 0.145 \\ -0.148 & -0.022 & 0.145 & 0.022 \end{bmatrix} \begin{matrix} su_9 \\ sv_9 \\ su_0 \\ sv_0 \end{matrix}$$

$$K_{17} = 1444 \times 10^9 \begin{bmatrix} u_9 & v_9 & u_0 & v_0 \\ 0.978 & 0.145 & -0.978 & -0.148 \\ 0.145 & 0.022 & -0.145 & -0.022 \\ -0.978 & -0.145 & 0.978 & 0.145 \\ -0.148 & -0.022 & 0.145 & 0.022 \end{bmatrix} \begin{matrix} u_9 \\ v_9 \\ u_0 \\ v_0 \end{matrix}$$

The global matrices are then assembled thus. Applying the boundary condition $su_0 = sv_0 = 0$, $K_1 + K_2 + K_3 + \dots K_{17}$ and formulating finite element, we have:

TABLE 1: GLOBAL MATRICES

		Global Matrix, $K = (k_1 + k_2 + k_3 + k_4 + k_5 + k_6 + k_7 + k_8 + k_9 + k_{10} + k_{11} + k_{12} + k_{13} + k_{14} + k_{15} + k_{16} + k_{17})$																			
K /		su_0	sv_0	su_1	sv_1	su_2	sv_2	su_3	sv_3	su_4	sv_4	su_5	sv_5	su_6	sv_6	su_7	sv_7	su_8	sv_8	su_9	sv_9
F_{1x}	$= 2.389 \times 10^{18}$	5.74	2.194	-0.98	-0.15	-0.733	-0.44	-0.332	-0.471	-0.054	-0.226	-1.948	-0.316	-0.721	-0.448	0	0	0	0	-0.98	-0.15
F_{1y}		2.19	2.254	-0.15	-0.02	-0.443	-0.27	-0.471	-0.667	-0.226	-0.945	-0.316	-0.052	-0.448	-0.279	0	0	0	0	-0.15	-0.02
F_{2x}		-0.98	-0.15	1.858	0.47	-0.88	-0.33	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{2y}		-0.15	-0.02	0.47	0.142	-0.325	-0.12	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{3x}		-0.73	-0.44	-0.88	-0.33	2.4	0.768	-0.787	-0.409	0	0	0	0	0	0	0	0	0	0	0	0
F_{3y}		-0.44	-0.27	-0.33	-0.12	1.177	0.387	-0.409	-0.213	0	0	0	0	0	0	0	0	0	0	0	0
F_{4x}		-0.33	-0.47	0	0	-0.787	0	1.291	1.258	-0.172	-0.378	0	0	0	0	0	0	0	0	0	0
F_{4y}		-0.47	-0.67	0	0	-0.409	0	1.258	1.708	-0.378	-0.828	0	0	0	0	0	0	0	0	0	0
F_{5x}		-0.05	-0.23	0	0	0	0	-0.172	-0.378	0.227	0.641	-0.001	-0.037	0	0	0	0	0	0	0	0
F_{5y}		-0.23	-0.95	0	0	0	0	-0.378	-0.828	0.641	2.771	-0.037	-0.998	0	0	0	0	0	0	0	0
F_{6x}		-1.95	-0.32	0	0	0	0	0	0	-0.001	-0.037	1.949	0.353	0	0	0	0	0	0	0	0
F_{6y}		-0.32	-0.05	0	0	0	0	0	0	-0.037	-0.998	0.353	1.05	0	0	0	0	0	0	0	0
F_{7x}		-0.72	-0.45	0	0	0	0	0	0	0	0	0	0	1.793	1.444	-1.07	-0.996	0	0	0	0
F_{7y}		-0.45	-0.28	0	0	0	0	0	0	0	0	0	0	1.444	1.207	-1	-0.928	0	0	0	0
F_{8x}		0	0	0	0	0	0	0	0	0	0	0	0	-1.072	-0.996	1.585	2.124	-0.51	-1.128	0	0
F_{8y}		0	0	0	0	0	0	0	0	0	0	0	0	-0.996	-0.928	2.124	3.418	-1.13	-2.49	0	0
F_{9x}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.51	-1.128	0.513	1.128	0	0
F_{9y}		0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1.13	-2.49	1.128	2.49	0	0
F_{10x}		-0.98	-0.15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.978	0.145
F_{10y}		-0.15	-0.02	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.145	0.022

B. Pressure consideration and Reservoir loading

The Reservoir under study is a cylinder shape with diameter 72.3m and height of 22m. There are 18 of such Reservoirs in Forcados terminal. The circular cross section of each storage tank was divided into 16 monitoring points distributed around the tank. These monitoring points were fixed at equal intervals on the outer face of the tanks and placed at 0.5m high from the tanks base. These monitoring

points were established in such a way that they remain fixed throughout the period of observation (Ehigiatior – Irughe, 2019).

During observations, the Reservoir is first filled to a high of 3m oil level, and three sets of Geomatics observations are carried out at this oil level, which are ovality, subsidence and verticality. The oil level is again filled to 10m, and the three sets of Geodetics observations are carried out as stated above.

The Reservoir is again filled to 19m, and the three sets of observations are conducted. For the purpose of this pilot Research project, we are only considering Reservoir filled to 19m oil level which is called Reservoir full oil level the pressure consideration for a reservoir at full oil level can be determined thus (Ehigiator – Irughe, 2019):

$$\left(\begin{array}{l} \text{Pressure} = \frac{\text{Force}}{\text{SurfaceArea}} = \frac{ma}{\text{Area}} \\ \text{Density} (\rho) = \frac{m}{v}, v = \text{volume} \\ \text{mass} = \rho \times v \end{array} \right) \quad (3)$$

$$\text{Vol. of cylinder} = \pi r^2 h = \pi \times 38.150 \times 19 = 86874.55 m^3$$

$$\text{Crude oil density} = 870 \text{ Kg/m}^3$$

$$\text{Mass} = 870 \times 86874.55 = 75,580,856.798 \text{ kg}$$

$$F = m \times a = 75,580,856.798 \times 9.8 = 740,692,396.619 \text{ N}$$

Surface Area of a cylinder

$$2\pi rh + 2\pi r^2 = 2\pi r(h+r) = 13,699.656 m^2$$

$$\text{Pressure} = \frac{F}{A} = \frac{740,692,396.619 \text{ N}}{13699.056} = 54,068.864 \text{ N} = 54.069 \times 10^3 \text{ N}$$

The pressure value so obtained shall be used to determine our deformation vector of the Reservoir under study. By substituting the pressure value $[U] = [F]$, and applying the boundary conditions and inverting the global matrices we have:

$$\left(\begin{array}{l} su_o = sv_o = 0, \quad f_{1x} = f_{1y} = 0, \quad f_{2x} = f_{2y} = f_{3x} = f_{3y}, f_{4x} = f_{4y}, f_{4x} = f_{4y}, f_{5x} = f_{5y}, f_{6x} = f_{6y}, \\ f_{7x} = f_{7y}, f_{8x} = f_{8y}, f_{9x} = f_{9y} = 54.069 \times 10^3 \text{ N} \end{array} \right)$$

TABLE 2: INVERSE OF GLOBAL MATRICES WITH BOUNDARY CONDITIONS

Global Matrix, $K = (k_1 + k_2 + k_3 + k_4 + k_5 + k_6 + k_7 + k_8 + k_9 + k_{10} + k_{11} + k_{12} + k_{13} + k_{14} + k_{15} + k_{16} + k_{17})$																			-1
K /	SU ₁	SV ₁	SU ₂	SV ₂	SU ₃	SV ₃	SU ₄	SV ₄	SU ₅	SV ₅	SU ₆	SV ₆	SU ₇	SV ₇	SU ₈	SV ₈	SU ₉	SV ₉	
SU ₁	1.86	0.47	-0.88	-0.33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
SV ₁	0.47	0.142	-0.33	-0.12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
SU ₂	-0.88	-0.33	2.4	0.768	-0.787	-0.41	0	0	0	0	0	0	0	0	0	0	0	0	
SV ₂	-0.33	-0.12	1.177	0.387	-0.409	-0.21	0	0	0	0	0	0	0	0	0	0	0	0	
SU ₃	0	0	-0.79	0	1.291	1.258	-0.172	-0.378	0	0	0	0	0	0	0	0	0	0	
SV ₃	0	0	-0.41	0	1.258	1.708	-0.378	-0.828	0	0	0	0	0	0	0	0	0	0	
SU ₄	0	0	0	-0.172	-0.38	0.2274	0.641	-0.001	-0.037	0	0	0	0	0	0	0	0	0	
SV ₄	0	0	0	-0.378	-0.83	0.641	2.771	-0.037	-0.998	0	0	0	0	0	0	0	0	0	
SU ₅	0	0	0	0	0	-0.001	-0.037	1.949	0.353	0	0	0	0	0	0	0	0	0	
SV ₅	0	0	0	0	0	-0.037	-0.998	0.353	1.05	0	0	0	0	0	0	0	0	0	
SU ₆	0	0	0	0	0	0	0	0	0	1.793	1.444	-1.072	-0.996	0	0	0	0	0	
SV ₆	0	0	0	0	0	0	0	0	0	1.444	1.207	-0.996	-0.928	0	0	0	0	0	
SU ₇	0	0	0	0	0	0	0	0	0	-1.072	-0.996	1.585	2.124	-0.51	-1.128	0	0	0	
SV ₇	0	0	0	0	0	0	0	0	0	-0.996	-0.928	2.124	3.418	-1.13	-2.49	0	0	0	
SU ₈	0	0	0	0	0	0	0	0	0	0	0	-0.513	-1.128	0.513	1.128	0	0	0	
SV ₈	0	0	0	0	0	0	0	0	0	0	0	-1.128	-2.49	1.128	2.49	0	0	0	
SU ₉	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.978	0.145	
SV ₉	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.145	0.022	

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The displacement vector for studs 1 to 9 are given as:

$$Displacement [D1] = \begin{bmatrix} su_1 \\ sv_1 \\ su_2 \\ sv_2 \\ su_3 \\ sv_3 \\ su_4 \\ sv_4 \\ su_5 \\ sv_5 \\ su_6 \\ sv_6 \\ su_7 \\ sv_7 \\ su_8 \\ sv_8 \\ su_9 \\ sv_9 \end{bmatrix} = \begin{bmatrix} 0.0120 \\ -0.0869 \\ -0.0560 \\ 0.0941 \\ -0.0918 \\ 0.0674 \\ 0.1276 \\ -0.0313 \\ 0.0044 \\ -0.0265 \\ -0.2522 \\ 0.4074 \\ -0.2632 \\ 0.4197 \\ -0.2014 \\ 0.3918 \\ -0.0567 \\ 0.3840 \end{bmatrix} \times 10^{-10}$$

The magnitude of displacement is given as:

$$[Magnitude] = [D1]^T \times [D1] = \begin{bmatrix} 0.0120 \\ -0.0869 \\ -0.0560 \\ 0.0941 \\ -0.0918 \\ 0.0674 \\ 0.1276 \\ -0.0313 \\ 0.0044 \\ -0.0265 \\ -0.2522 \\ 0.4074 \\ -0.2632 \\ 0.4197 \\ -0.2014 \\ 0.3918 \\ -0.0567 \\ 0.3840 \end{bmatrix} \times 10^{-10} \times \begin{bmatrix} 0.0120 \\ -0.0869 \\ -0.0560 \\ 0.0941 \\ -0.0918 \\ 0.0674 \\ 0.1276 \\ -0.0313 \\ 0.0044 \\ -0.0265 \\ -0.2522 \\ 0.4074 \\ -0.2632 \\ 0.4197 \\ -0.2014 \\ 0.3918 \\ -0.0567 \\ 0.3840 \end{bmatrix} \times 10^{-10} = [7.1507e-47]$$

C. Strain analysis

When an object is subjected to external force, initially at equilibrium condition, the deformation effect is negligible. But with a gradual increase in the magnitude of the applied load, the dimensions of the object will be altered is called d due to deformation. The ratio of change in the dimension due to deformation of the object to its original dimension is called strain. Strain is defined as the force tending to pull or stretch an object to an extreme or damaging degree. The finite element model for strain analysis is given as:

$$\begin{bmatrix} \xi_x \\ \xi_y \\ \xi_{xy} \end{bmatrix} = [B] \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix} \quad (4)$$

Where; [B] is as defined in equation (3), are the deformation at nodes 1, 2 and 3 respectively. By substituting the derived values in table 2 into equation (2) and (4), we have:

$$\begin{bmatrix} \xi_x \\ \xi_y \\ \xi_{xy} \end{bmatrix} = [B] \begin{bmatrix} su_0 \\ sv_0 \\ su_1 \\ sv_1 \\ su_2 \\ sv_2 \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} \beta_1 & 0 & \beta_2 & 0 & \beta_3 & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & \beta_1 & \gamma_2 & \beta_2 & \gamma_3 & \beta_3 \end{bmatrix} \begin{bmatrix} su_0 \\ sv_0 \\ su_1 \\ sv_1 \\ su_2 \\ sv_2 \end{bmatrix} \quad (5)$$

Where

$$[B] = \frac{1}{2A} \begin{bmatrix} \beta_1 & 0 & \beta_2 & 0 & \beta_3 & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & \beta_1 & \gamma_2 & \beta_2 & \gamma_3 & \beta_3 \end{bmatrix}$$

where

$$\beta_1 = y_2 - y_3, \beta_2 = y_3 - y_1, \beta_3 = y_1 - y_2$$

$$\gamma_1 = x_2 - x_3, \gamma_2 = x_3 - x_1, \gamma_3 = x_1 - x_2$$

or

$$\beta_1 = y_{23}, \beta_2 = y_{31}, \beta_3 = y_{12}$$

$$\gamma_1 = x_{23}, \gamma_2 = x_{31}, \gamma_3 = x_{12}$$

$$\begin{bmatrix} \xi_{su1} \\ \xi_{sv1} \\ \xi_{su1} \end{bmatrix} = \frac{1}{2 \times 280.577} \begin{bmatrix} -14.072 & 0 & 19.671 & 0 & -5.599 & 0 \\ 0 & 5.833 & 0 & 32.593 & 0 & -38.426 \\ 5.833 & -14.072 & 32.593 & 19.671 & -38.426 & -5.599 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -0.012 \\ -0.0869 \\ -0.0560 \\ 0.0941 \end{bmatrix} \times 10^{-10}$$

$$\begin{bmatrix} \xi_{su1} \\ \xi_{sv1} \\ \xi_{su1} \end{bmatrix} = \begin{bmatrix} 0.0014 \\ -0.1149 \\ -0.0085 \end{bmatrix} \times 10^{-11}$$

D. Stress expression

The intensity of the resisting force offered by the inter molecular bonding of the object per unit area towards the load applied before the failure of the object is called as stress. This is also defined as the force per cross – sectional Area and the modules of elasticity for this material under study is given as;

Using modules of elasticity of $210 \text{ Gpa} = 210 \times 1000 = 210,000 \text{ N/mm}^2$ and the poison value to be 0.25, the equivalent stress expression is given by equation (6)

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} = D \begin{bmatrix} \xi_x \\ \xi_y \\ \xi_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \xi_x \\ \xi_y \\ \xi_{xy} \end{bmatrix} \quad (6)$$

The stress expressions at node 1 are determined by substituting the values of the strain at node 1 into equation (6), we have:

$$\begin{bmatrix} \xi_{su2} \\ \xi_{sv2} \\ \xi_{su2} \end{bmatrix} = \frac{1}{2 \times 290.855} \begin{bmatrix} -11.362 & 0 & 31.033 & 0 & -19.671 & 0 \\ 0 & 10.719 & 0 & 21.874 & 0 & -32.593 \\ 10.719 & -11.362 & 21.874 & 31.033 & -19.671 & -32.593 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -0.0560 \\ 0.0941 \\ -0.0918 \\ 0.0674 \end{bmatrix} \times 10^{-10}$$

The Strain displacement parameters for the Truss element 1 are given in Table 3 below.

TABLE 3: TRUSS ELEMENT 1

Truss _1						
Studs	Y(m)	X(m)	Strain Displacement Parameters			
Center	840.028	581.201	β_1	-14.072	γ_1	5.833
1	845.627	619.627	β_2	19.671	γ_2	32.593
2	859.699	613.794	β_3	-5.599	γ_3	-38.426

By substituting the values in table 3 and the equivalent displacement values from finite element formulation into equation (5) we have:

$$\begin{bmatrix} \sigma_{su1} \\ \sigma_{sv1} \\ \sigma_{su1} \end{bmatrix} = \begin{bmatrix} -0.0765 \\ -0.3207 \\ -0.0238 \end{bmatrix} \times 10^{-6}$$

The Strain displacement parameters for the Truss element 2 are given in table 1 below.

TABLE 4: TRUSS ELEMENT 2

Truss _2						
Stud	Y	X	Strain Displacement Parameters			
Center	840.028	581.201	β_1	-11.362	γ_1	10.719
2	859.699	613.794	β_2	31.033	γ_2	21.874
3	871.061	603.075	β_3	-19.671	γ_3	-32.593

By substituting the values in table 2 and the equivalent displacement values from finite element formulation into equation (2) we have:

$$\begin{bmatrix} \xi_{su2} \\ \xi_{sv2} \\ \xi_{suv2} \end{bmatrix} = \begin{bmatrix} 0.0117 \\ -0.0238 \\ 0.2242 \end{bmatrix} \times 10^{-12}$$

The Strain displacement parameters for the Truss element 3 are given in Table 5 below.

TABLE 5: TRUSS ELEMENT 3

Truss _3						
Stud	Y	X	Strain Displacement Parameters			
Center	840.028	581.201	β_1	-5.935	γ_1	13
3	871.061	603.075	β_2	36.968	γ_2	8.874
4	876.996	590.075	β_3	-31.033	γ_3	-21.874

By substituting the values in table 3 and the equivalent displacement values from finite element formulation into equation (2) we have:

$$\begin{bmatrix} \xi_{su3} \\ \xi_{sv3} \\ \xi_{suv3} \end{bmatrix} = \frac{1}{2 \times 266.398} \begin{bmatrix} -5.935 & 0 & 36.968 & 0 & -31.033 & 0 \\ 0 & 13 & 0 & 8.874 & 0 & -21.874 \\ 13 & -5.935 & 8.874 & 36.968 & -31.033 & -21.874 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -0.0918 \\ 0.00674 \\ 0.1276 \\ -0.0313 \end{bmatrix} \times 10^{-10}$$

$$\begin{bmatrix} \xi_{su3} \\ \xi_{sv3} \\ \xi_{suv3} \end{bmatrix} = \begin{bmatrix} -0.1380 \\ 0.0140 \\ -0.0721 \end{bmatrix} \times 10^{-11}$$

The stress values at node 3 are determined as:

$$\begin{bmatrix} \sigma_{su3} \\ \sigma_{sv3} \\ \sigma_{suv3} \end{bmatrix} = \begin{bmatrix} 0.0004 \\ -0.0153 \\ 0.1758 \end{bmatrix} \times 10^{-5}$$

The Strain displacement parameters for the Truss element 4 are given in table 6 below.

TABLE 6: TRUSS ELEMENT 4

Truss _4						
Stud	Y	X	Strain Displacement Parameters			
Center	840.028	581.201	β_1	-0.548	γ_1	14.938
4	876.996	590.075	β_2	37.516	γ_2	-6.064
5	877.544	575.137	β_3	-36.968	γ_3	-8.874

By substituting the values in table 4 and the equivalent displacement values from finite element formulation into equation (1 – 2) we have:

$$\begin{bmatrix} \xi_{su4} \\ \xi_{sv4} \\ \xi_{suv4} \end{bmatrix} = \frac{1}{2 \times 279.298} \begin{bmatrix} -0.548 & 0 & 37.516 & 0 & -36.968 & 0 \\ 0 & 14.938 & 0 & -6.064 & 0 & -8.864 \\ 14.938 & -0.548 & 14.938 & 37.516 & -36.968 & -8.864 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0.1276 \\ -0.0313 \\ 0.0044 \\ -0.0265 \end{bmatrix} \times 10^{-10}$$

$$\begin{bmatrix} \xi_{su4} \\ \xi_{sv4} \\ \xi_{suv4} \end{bmatrix} = \begin{bmatrix} 0.8279 \\ 0.0760 \\ 0.1439 \end{bmatrix} \times 10^{-12}$$

The equivalent stress expression for node 4 are given below:

$$\begin{bmatrix} \sigma_{su4} \\ \sigma_{sv4} \\ \sigma_{suv4} \end{bmatrix} = \begin{bmatrix} 0.2371 \\ 0.0792 \\ 0.0403 \end{bmatrix} \times 10^{-6}$$

TABLE 7: TRUSS ELEMENT 5

Truss _5						
Stud	Y	X	Strain Displacement Parameters			
0	840.028	581.201	β_1	5.173	γ_1	14.766
5	877.544	576.137	β_2	32.343	γ_2	-19.83
6	872.371	561.371	β_3	-37.516	γ_3	5.064

By substituting the values in table 5 and the equivalent displacement values from finite element formulation into equation (2) we have:

$$\begin{bmatrix} \xi_{su5} \\ \xi_{sv5} \\ \xi_{suv5} \end{bmatrix} = \frac{1}{2 \times 280.118} \begin{bmatrix} 5.173 & 0 & 32.343 & 0 & -37.516 & 0 \\ 0 & 14.766 & 0 & -19.830 & 0 & 5.064 \\ 14.766 & 5.173 & -19.830 & 32.343 & 5.064 & -37.517 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0.0044 \\ -0.0265 \\ -0.2522 \\ 0.4074 \end{bmatrix} \times 10^{-10}$$

$$\begin{bmatrix} \xi_{su5} \\ \xi_{sv5} \\ \xi_{suv5} \end{bmatrix} = \begin{bmatrix} 0.1714 \\ 0.0462 \\ -0.3125 \end{bmatrix} \times 10^{-11}$$

Stress at node 5 are determined thus:

$$\begin{bmatrix} \sigma_{su5} \\ \sigma_{sv5} \\ \sigma_{suv5} \end{bmatrix} = \begin{bmatrix} 0.5123 \\ 0.2493 \\ -0.8750 \end{bmatrix} \times 10^{-6}$$

By substituting the values in table 6 and the equivalent displacement values from finite element formulation into equation (2) we have:

TABLE 8: TRUSS ELEMENT 6

Truss _6						
Stud	Y	X	Strain Displacement Parameters			
Center	840.028	581.201	β_1	10.178	γ_1	10.938
6	872.371	561.098	β_2	22.165	γ_2	-31.041
7	862.193	550.16	β_3	-32.343	γ_3	20.103

$$\begin{bmatrix} \xi_{su6} \\ \xi_{sv6} \\ \xi_{suv6} \end{bmatrix} = \frac{1}{2 \times 278.043} \begin{bmatrix} 10.178 & 0 & 22.165 & 0 & -32.343 & 0 \\ 0 & 10.938 & 0 & -31.041 & 0 & 20.103 \\ 10.938 & 10.178 & -31.041 & 22.165 & -32.343 & -37.517 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -0.2522 \\ 0.4074 \\ -0.2632 \\ 0.4197 \end{bmatrix} \times 10^{-10}$$

$$\begin{bmatrix} \xi_{su6} \\ \xi_{sv6} \\ \xi_{suv6} \end{bmatrix} = \begin{bmatrix} 0.0526 \\ -0.0757 \\ 1.731 \end{bmatrix} \times 10^{-11}$$

The values for the stress at node 6 are as follows:

$$\begin{bmatrix} \sigma_{su6} \\ \sigma_{sv6} \\ \sigma_{suv6} \end{bmatrix} = \begin{bmatrix} 0.0094 \\ -0.0175 \\ 0.4847 \end{bmatrix} \times 10^{-5}$$

By substituting the values in table 7 and the equivalent displacement values from finite element formulation into equation (2) we have:

TABLE 9: TRUSS ELEMENT 7

Truss _7						
Stud	Y	X	Strain Displacement Parameters			
Center	840.028	581.201	β_1	13.667	γ_1	6.197
7	862.193	550.16	β_2	8.498	γ_2	-37.238
8	848.526	543.963	β_3	-22.165	γ_3	31.041

$$\begin{bmatrix} \xi_{su7} \\ \xi_{sv7} \\ \xi_{suv7} \end{bmatrix} = \frac{1}{2 \times 276.175} \begin{bmatrix} 13.667 & 0 & 8.498 & 0 & -22.165 & 0 \\ 0 & 6.197 & 0 & -37.238 & 0 & 31.041 \\ 6.197 & 13.667 & -37.238 & 8.498 & 31.041 & -22.165 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -0.2632 \\ 0.4197 \\ -0.2014 \\ 0.3918 \end{bmatrix} \times 10^{-10}$$

$$\begin{bmatrix} \xi_{su7} \\ \xi_{sv7} \\ \xi_{suv7} \end{bmatrix} = \begin{bmatrix} 0.4033 \\ -0.6277 \\ -0.2839 \end{bmatrix} \times 10^{-12}$$

The stress values at node 7 are determined thus:

$$\begin{bmatrix} \sigma_{su7} \\ \sigma_{sv7} \\ \sigma_{suv7} \end{bmatrix} = \begin{bmatrix} 0.0014 \\ -0.0042 \\ 0.1357 \end{bmatrix} \times 10^{-6}$$

$$\begin{bmatrix} \xi_{su8} \\ \xi_{sv8} \\ \xi_{suv8} \end{bmatrix} = \frac{1}{2 \times 262.662} \begin{bmatrix} 14.098 & 0 & -5.600 & 0 & -8.498 & 0 \\ 0 & 0.566 & 0 & -37.804 & 0 & 37.238 \\ 0.566 & 14.098 & -37.804 & -5.600 & 37.238 & -8.498 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -0.2014 \\ 0.3918 \\ -0.0567 \\ 0.3840 \end{bmatrix} \times 10^{-10}$$

$$\begin{bmatrix} \xi_{su8} \\ \xi_{sv8} \\ \xi_{suv8} \end{bmatrix} = \begin{bmatrix} 0.3064 \\ -0.0975 \\ 0.0086 \end{bmatrix} \times 10^{-12}$$

The equivalent stress at node 8 are given as:

$$\begin{bmatrix} \sigma_{su8} \\ \sigma_{sv8} \\ \sigma_{suv8} \end{bmatrix} = \begin{bmatrix} 0.7897 \\ -0.0585 \\ 0.0241 \end{bmatrix} \times 10^{-7}$$

E. Statistical Analysis

The global matrices obtained are Normal equation. A normal equation is that equation that is equal to their

$$[\sigma^2] = \begin{bmatrix} 0.0120 \\ -0.0869 \\ -0.0560 \\ 0.0941 \\ -0.0918 \\ 0.0674 \\ 0.1276 \\ -0.0313 \\ 0.0044 \\ -0.0265 \\ -0.2522 \\ 0.4074 \\ -0.2632 \\ 0.4197 \\ -0.2014 \\ 0.3918 \\ -0.0567 \\ 0.3840 \end{bmatrix}^T \times \begin{bmatrix} 0.0120 \\ -0.0869 \\ -0.0560 \\ 0.0941 \\ -0.0918 \\ 0.0674 \\ 0.1276 \\ -0.0313 \\ 0.0044 \\ -0.0265 \\ -0.2522 \\ 0.4074 \\ -0.2632 \\ 0.4197 \\ -0.2014 \\ 0.3918 \\ -0.0567 \\ 0.3840 \end{bmatrix}$$

9

$$\sigma^2 = 1.526 \times 10^{-25}$$

The variance covariance matrix of displacement parameters CL is given as:

$$CL = \sigma^2 [N]^{-1} \quad (8)$$

TABLE 10: TRUSS ELEMENT 8

Truss _8						
Stud	Y	X	Strain Displacement Parameters			
Center	840.028	581.201	β_1	14.098	γ_1	0.566
8	848.526	543.963	β_2	-5.6	γ_2	-37.804
9	834.428	543.397	β_3	-8.498	γ_3	37.238

By substituting the values in table 8 and the equivalent displacement values from finite element formulation into equation (1 – 2) we have:

transpose.

The variance matrix of the displacement vector is given as:

$$\sigma^2 = \frac{Suv_i^T \times Suv_i}{r} \quad (7)$$

Where r = redundancy, which is equal to the differences between numbers of observation and the numbers of monitoring station. Number of displacements 18, number of studs 9, therefore, r = 9.

Where N is the global matrices:

TABLE 11: VARIANCE COVARIANCE MATRIX OF DISPLACEMENT PARAMETERS

$CL =$	5.397e-25	-1.910e-24	-3.421e-26	-5.426e-26	-1.953e-26	2.712e-27	4.308e-27	-9.368e-28	-8.101e-28	3.350e-27	8.670e-29	-1.155e-28	-4.220e-28	-2.662e-29	-4.283e-28	-1.078e-28	-4.559e-29	8.269e-30
	-1.910e-24	8.249e-24	2.344e-25	3.711e-25	1.335e-25	-1.854e-26	-2.946e-26	6.406e-27	5.540e-27	-2.291e-26	-5.929e-28	7.9008e-28	2.886e-27	1.820e-28	2.929e-27	7.377e-28	3.117e-28	-5.654e-29
	-3.421e-26	2.344e-25	-3.789e-26	2.460e-25	8.859e-26	-1.230e-26	-1.953e-26	4.248e-27	3.674e-27	-1.519e-26	-3.932e-28	5.239e-28	1.914e-27	1.207e-28	1.942e-27	4.892e-28	2.067e-28	-3.750e-29
	-5.426e-26	3.711e-25	2.460e-25	-4.399e-25	-1.583e-25	2.198e-26	3.492e-26	-7.595e-27	-6.567e-27	2.716e-26	7.028e-28	-9.365e-28	-3.421e-27	-2.158e-28	-3.472e-27	-8.745e-28	-3.696e-28	6.703e-29
	1.260e-27	-8.619e-27	-5.715e-27	1.021e-26	-1.397e-25	2.583e-25	-1.352e-25	2.911e-26	2.551e-26	-1.055e-25	-2.730e-27	3.638e-27	1.329e-26	8.383e-28	1.348e-26	3.396e-27	1.435e-27	-2.603e-28
	-3.721e-26	2.545e-25	1.687e-25	-3.017e-25	4.247e-25	-5.177e-25	2.252e-25	-4.841e-26	-4.250e-26	1.757e-25	4.548e-27	-6.061e-27	-2.214e-26	-1.396e-27	-2.246e-26	-5.659e-27	-2.391e-27	4.338e-28
	-3.676e-26	2.514e-25	1.667e-25	-2.980e-25	4.593e-25	-5.878e-25	1.765e-25	-1.451e-25	-7.530e-27	3.113e-26	8.058e-28	-1.073e-27	-3.923e-27	-2.474e-28	-3.980e-27	-1.002e-27	-4.237e-28	7.685e-29
	-4.411e-26	3.016e-25	2.000e-25	-3.576e-25	4.948e-25	-5.971e-25	1.696e-25	1.260e-26	-4.378e-26	1.810e-25	4.686e-27	-6.244e-27	-2.281e-26	-1.438e-27	-2.314e-26	-5.830e-27	-2.464e-27	4.469e-28
	-3.888e-28	2.659e-27	1.763e-27	-3.152e-27	-3.044e-27	8.958e-27	3.682e-26	6.070e-27	8.785e-26	2.244e-27	2.362e-26	-3.227e-26	-1.101e-25	-7.482e-27	-1.119e-25	-2.821e-26	-1.192e-26	2.162e-27
	-7.221e-26	4.938e-25	3.275e-25	-5.854e-25	8.551e-25	-1.064e-24	2.946e-25	-1.211e-25	-8.206e-26	3.258e-25	7.558e-27	-1.004e-26	-3.697e-26	-2.312e-27	-3.751e-26	-9.447e-27	-3.992e-27	7.241e-28
	-6.627e-27	4.532e-26	3.005e-26	-5.372e-26	7.643e-26	-9.376e-26	3.642e-26	-9.324e-27	1.600e-26	3.002e-26	1.350e-25	-2.505e-25	-2.267e-25	-6.183e-26	-2.499e-25	-6.382e-26	-2.697e-26	4.892e-27
	8.991e-27	-6.149e-26	-4.077e-26	7.289e-26	-1.036e-25	1.271e-25	-4.951e-26	1.263e-26	-2.193e-26	-4.074e-26	-2.505e-25	5.345e-25	-7.224e-27	1.348e-25	4.933e-26	1.4863e-26	6.281e-27	-1.139e-27
	3.128e-26	-2.139e-25	-1.418e-25	2.535e-25	-3.609e-25	4.426e-25	-1.713e-25	4.411e-26	-7.417e-26	-1.417e-25	-2.266e-25	-7.480e-27	-6.961e-25	-4.906e-26	-7.085e-25	-1.785e-25	-7.546e-26	1.368e-26
	2.081e-27	-1.423e-26	-9.438e-27	1.687e-26	-2.399e-26	2.942e-26	-1.146e-26	2.923e-27	-5.090e-27	-9.430e-27	-6.183e-26	1.348e-25	-4.900e-26	1.447e-25	1.220e-26	5.738e-27	2.425e-27	-4.398e-28
	3.178e-26	-2.173e-25	-1.441e-25	2.575e-25	-3.667e-25	4.497e-25	-1.741e-25	4.482e-26	-7.543e-26	-1.440e-25	-2.498e-25	4.908e-26	-7.085e-25	1.213e-26	-1.282e-24	2.736e-25	1.156e-25	-2.097e-26
	8.007e-27	-5.476e-26	-3.631e-26	6.491e-26	-9.239e-26	1.133e-25	-4.388e-26	1.129e-26	-1.900e-26	-3.628e-26	-6.379e-26	1.480e-26	-1.785e-25	5.722e-27	2.736e-25	-2.008e-25	-8.486e-26	1.539e-26
	3.384e-27	-2.314e-26	-1.534e-26	2.743e-26	-3.904e-26	4.788e-26	-1.854e-26	4.772e-27	-8.033e-27	-1.533e-26	-2.696e-26	6.255e-27	-7.545e-26	2.418e-27	1.156e-25	-8.486e-26	1.253e-25	-2.273e-26
	-6.138e-28	4.197e-27	2.783e-27	-4.976e-27	7.082e-27	-8.685e-27	3.363e-27	-8.656e-28	1.456e-27	2.781e-27	4.890e-27	-1.134e-27	1.368e-26	-4.386e-28	-2.097e-26	1.539e-26	-2.273e-26	1.537e-25

Variance covariance of the observation are given as:

$$CLL = [N][CL][N]^{-1} \quad (9)$$

TABLE 12: VARIANCE COVARIANCE OF THE OBSERVATIONS

$CLL =$	2.835e-25	7.172e-26	-1.342e-25	-4.959e-26	5.068e-41	1.591e-40	1.1591e-40	1.862e-41	-4.845e-41	-8.558e-42	3.000e-42	1.098e-42	-6.713e-44	-2.028e-43	-1.516e-44	-5.554e-45	-1.469e-43	-2.275e-45
	7.172e-26	2.166e-26	-4.959e-26	-1.831e-26	1.870e-41	4.458e-41	3.069e-41	3.954e-42	-1.298e-41	-2.750e-42	8.938e-43	3.538e-43	-6.749e-44	-9.297e-44	1.602e-44	5.273e-44	-5.107e-44	-1.522e-45
	-1.342e-25	-4.959e-26	3.662e-25	1.7961e-25	-1.097e-40	-1.370e-40	-8.859e-42	1.569e-41	8.059e-42	2.689e-41	-4.532e-42	-1.788e-42	2.388e-43	5.126e-43	-4.426e-44	-3.329e-43	3.257e-43	9.099e-45
	-4.959e-26	-1.831e-26	1.7961e-25	5.905e-26	-6.241e-26	-3.250e-26	5.501e-41	1.125e-41	-1.735e-41	2.520e-42	-7.701e-42	-1.209e-41	2.199e-42	2.208e-42	-1.337e-43	-2.392e-43	7.580e-43	3.276e-44
	7.373e-42	3.431e-43	2.928e-41	-6.241e-26	1.970e-25	1.919e-25	-2.624e-26	-5.768e-26	2.168e-41	2.707e-41	1.290e-41	5.076e-41	-1.206e-41	-1.322e-41	1.177e-42	2.492e-42	-2.879e-42	-1.679e-43
	3.508e-41	1.011e-41	-4.853e-41	-3.250e-26	1.919e-25	2.606e-25	2.606e-25	-5.768e-26	-1.263e-25	8.586e-41	1.073e-43	6.023e-41	-1.794e-41	-1.902e-41	2.078e-43	8.969e-44	-3.626e-42	-2.499e-43
	-1.043e-40	-3.342e-41	1.086e-40	3.098e-41	-2.624e-26	-5.768e-26	3.464e-26	-1.526e-28	-5.646e-27	-1.522e-25	1.117e-40	4.874e-41	3.590e-42	-1.487e-41	9.622e-43	4.525e-42	-7.862e-43	-1.024e-43
	-4.214e-41	-1.085e-41	-2.128e-41	-6.9474e-41	-5.768e-26	-1.263e-25	-1.526e-28	2.965e-25	-5.646e-27	-1.522e-25	6.496e-41	-4.808e-41	2.789e-41	1.622e-41	1.609e-42	6.202e-42	3.145e-42	4.283e-44
	5.907e-41	1.961e-41	-1.001e-40	1.993e-42	-1.692e-40	-1.976e-40	-5.646e-27	-5.646e-27	2.853e-25	7.660e-26	-1.365e-25	-4.684e-26	2.655e-42	7.293e-42	-4.934e-43	5.138e-42	-2.075e-42	1.710e-43
	7.708e-41	2.200e-41	-6.1409e-41	1.247e-41	-1.606e-40	-1.913e-40	-1.522e-25	-1.522e-25	7.660e-26	1.724e-25	-4.684e-26	-1.617e-26	-1.110e-41	1.394e-42	-4.272e-42	-1.099e-41	-3.164e-42	-2.701e-43
	-1.878e-41	-5.563e-42	1.56e-41	2.279e-42	7.355e-42	4.827e-41	1.565e-40	5.058e-41	-1.365e-25	-4.684e-26	3.283e-25	1.532e-25	-8.179e-26	-3.799e-26	3.074e-41	8.329e-41	2.349e-41	-2.595e-43
	-1.838e-41	-5.574e-42	2.036e-41	-1.065e-42	2.952e-41	3.750e-41	3.424e-41	1.766e-42	-4.684e-26	-1.617e-26	1.532e-25	1.295e-25	-3.799e-26	-7.080e-26	1.122e-41	2.984e-41	7.186e-42	7.177e-44
	-9.219e-43	1.108e-43	3.899e-42	7.087e-42	-3.405e-41	-4.792e-41	-6.057e-41	-1.475e-41	2.286e-41	1.691e-42	-8.179e-26	-3.799e-26	8.179e-26	3.799e-26	-5.737e-26	-1.266e-25	-2.896e-41	1.170e-42
	1.545e-42	5.452e-43	-5.018e-43	-8.037e-43	-1.951e-42	-1.442e-41	-4.637e-41	-1.011e-41	6.700e-42	-4.133e-43	-3.799e-26	-7.080e-26	3.799e-26	2.278e-25	-2.609e-26	-5.600e-26	-1.341e-41	-4.728e-43
	-1.557e-42	-6.053e-43	-1.198e-42	4.9764e-42	-2.407e-42	1.203e-41	6.734e-41	1.544e-41	-2.657e-41	-9.826e-42	-1.763e-41	-1.020e-41	-5.737e-26	-2.609e-26	3.387e-26	9.644e-26	1.461e-41	9.897e-43
	7.774e-43	6.145e-43	-1.782e-41	-2.101e-42	-9.363e-42	6.876e-42	6.925e-41	2.634e-41	1.105e-41	8.574e-42	-1.194e-40	-5.748e-41	-1.266e-25	-5.600e-26	9.644e-26	1.522e-25	-6.104e-26	-1.182e-42
	2.697e-42	8.150e-43	-1.865e-42	-3.036e-42	7.681e-42	7.814e-42	-9.332e-43	-3.510e-42	2.260e-42	1.681e-42	-1.025e-41	-4.546e-42	-6.719e-42	-3.652e-42	7.316e-42	-6.104e-26	1.495e-25	2.823e-26
	-1.757e-44	3.228e-44	-7.031e-43	2.229e-43	-1.908e-42	-1.927e-42	4.164e-43	1.998e-43	2.868e-43	-4.068e-43	-2.899e-42	-1.338e-42	2.979e-42	1.250e-42	-1.700e-42	-4.162e-42	2.823e-26	1.556e-25

The cofactor matrices of the displacement values are given as:

$$QV = [1 - A \times (A^T \times A)^{-1} \times A^T] \quad (10)$$

TABLE 13: THE COFACTOR MATRICES OF THE DISPLACEMENT VECTORS

$QV =$	-1.78e-15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.00	1.16e-13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.00	1.00	-2.44e-15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.00	1.00	1.00	4.88e-15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.00	1.00	1.00	1.00	-1.33e-15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.00	1.00	1.00	1.00	1.00	-4.44e-15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.00	1.00	1.00	1.00	1.00	1.00	-2.11e-14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	7.77e-15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	3.11e-15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	2.48e-14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.55e-15	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	3.33e-15	1.00	1.00	1.00	1.00	1.00	1.00
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	3.66e-15	1.00	1.00	1.00	1.00	1.00
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-2.22e-16	1.00	1.00	1.00	1.00
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	3.55e-15	1.00	1.00	1.00
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.33e-15	1.00	1.00
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	3.33e-16	1.00
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.11e-16

IV. CONCLUSION

This Research is to consolidate our pilot project for a Reservoir with 72.3 m diameter and height of 22 m surrounded with 16 studs serving as monitoring stations. In this pilot phase II paper, we used only nine (9) studs, i.e. stud 1 to 9 and the center of the Reservoir with 38.15 m as the radius. For the purpose of the paper, the Reservoir was divided into nine parts. Based on the presented analysis, the following results can be summarized:

1. We have demonstrated that it is possible to use finite Element analysis model in deformation of large surface Reservoir.

2. The precision and accuracy increased with increased number of points under study.

3. The suggested technique of analysis the structural deformation observations can be used to identify and determine the values of deformation for any structure for one single epoch of observations.

3. The magnitude displacement vectors was $7.1507e-47$.

4. The variance matrix displacement vectors was 1.526×10^{-25} .

A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

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