ORIGINAL ARTICLE

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Adjustment of Single and Double Braced Quadrilateral using Least Square Condition Equation

R. Ehigiator-Irughe and B. M. Mohammad Muhajir

ABSTRACT

Quadrilateral is a single or double triangulation system consisting of figures with four corners stations. It consists of two known stations and a base line. Other corners of the triangles are measured using precise instrument. The systems is treated as the strongest with the best arranged triangular structure which provides adequate means of determining the lengths of other sides of the triangle whose length, bearings and positions are required. In this four sided polygon with four (4) points were established. The coordinate of two points (A and B) are known while the coordinates of points (C and D) are required. The purpose of the exercise was to use least square condition equation to determine and adjust coordinates of the two unknown points using the Angular measurements of quadrilateral. Two separate measurements were taken (observation 1 and observation 2) forming a network of a single and double braced quadrilateral respectively. After which, the data obtained were reduced and then adjusted using least square condition equation.

Keywords: Quadrilateral, Condition Equation, Observation, normal equation, Lagrange.

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I. Introduction

Horizontal control has been performed using our classical Geomatics Engineering approaches. Some of the approaches include but not limited to; Traversing, Trilateration, Triangulation and recently Global navigation Satellite systems (GNSS). One of the methods of triangulation is the subdivision of complex figure into smaller triangles as either single or double quadrilateral. The purpose is to determine the coordinates of unknown points using least square method.

Geomatics measurements are usually inseparable with errors during field observations and these errors are due to many factors, which include but not limited to human, natural and equipment used etc. The errors cannot be left in the final data without finding a way to eliminate them; this gave reason for the use of mathematical adjustment procedure [1]. In the first half of the 19th century the Least Squares (LS) adjustment technique was developed [2]. LS are the conventional technique for adjusting Geomatics measurements till date. The LS technique minimizes the sum of the squares of differences between the observation and estimate [3]. Apart from LS other methods of adjusting Geomatics methods have been developed, such as Kalman Filter (KF) [4], Least Squares Collocation (LSC) [5] and Total Least Squares (TLS) [6] – [9]. This work will expound in its simplest form fundamental methods of LS adjustment equation as it applies to single and double Quadrilateral Triangular figure using condition equation method.

II. Conditional Least Squares Adjustment

The general form of the Conditional mathematical model given by:

$$B_{r,n}v_{n,1} + \Delta_{r,1} = 0, \qquad (1)$$

where B is the deigned matrix,
V is the vector,
Dimension r is the redundancy,
dimension n is the number of given observations
Δ is the vector of misclosure.

In equation (1) *r (degrees of freedom)=n–no.*

Two basic properties must be satisfied for the conditional model:

1) Number of equations = Number of degrees of freedoms. This means that each redundant observation provides one independent condition equation.

2) The equations describe the functional relationship among the observations only. This obviously indicates that the unknown parameters x will not be among the direct output of the conditional adjustment. Thus the adjusted parameters X^ and their covariance matrix C X^ , have to be computed after the adjustment using the direct model (X^ = f (^I)) and the law of propagation of variances (this is a disadvantage when comparing the conditional and the parametric adjustment).

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A. Adjustment of a single braced quadrilateral using least square condition equation

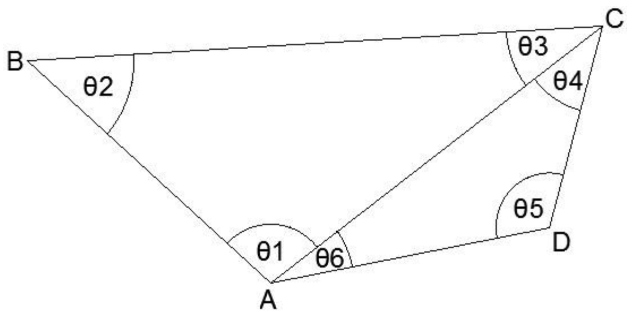

Fig. 1: Single Braced Quadrilateral.

The field measurements for a single braced quadrilateral ABCD are given below. Using this data, we will compute the coordinates of station C and D.

TABLE 1: FIELD OBSERVATION DATA

ANGLE (θ)	STN FROM	FACE	Degree	mins	sec	STN TO
1	A	L	00	00	00	D
		L	19	11	23	C
		R	199	11	24	C
		R	180	00	02	D
6	A	L	0	00	00	C
		L	29	58	13	B
		R	209	58	12	B
		R	179	59	58	C
2	B	L	0	00	00	D
		L	117	37	01	A
		R	297	37	03	A
		R	180	00	01	D
3	B	L	0	00	00	D
		L	43	11	22	C
		R	223	11	29	C
		R	179	59	59	D
4	C	L	0	00	00	A
		L	48	21	31	B
		R	228	21	33	B
		R	180	00	03	A
5	C	L	0	00	00	A
		L	101	40	08	D
		R	281	40	07	D
		R	180	00	01	A

TABLE 3: OBSERVED ANGLES AND STANDARD ERRORS

ANGLE (θ)	OBS. VALUE (θ)	STD ERROR (σ^2)	Station	Eastings	Northings
1	19° 11' 22.5"	0.7	A	507327.641	91756.439
2	117° 37' 1.5"	0.7	B	507450.410	92079.322
3	43° 11' 26.0"	5.7			
4	48° 21' 30.5"	0.7			
5	101° 40' 7.0"	1.4			
6	29° 58' 13.5"	0.7			

The number of total observations (n) = 6 Angles.
No of sides (S) = 4.
The number of necessary observations (n_0) = 2 (No of new points) = 2(S-2) = 4 Hence, the number of conditions (r) = n –n_0 = 2.

1. Condition equations

There are two condition equation:

V1+V2+V3=180°–(θ1+θ2+θ3) = 180°-179°59'50.0"=Δ1
Δ1=10.0"
V4+V5+V6=180°–(θ4+θ5+θ6) = 180°-179°59'51.0"=Δ2,
Δ2=9.0"

The design matrix is given as:

$$B = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

The vector is given as:

$$v = \begin{bmatrix} v1 \\ v2 \\ v3 \\ v4 \\ v5 \\ v6 \end{bmatrix}$$

The general for is given as:

$$[B][V] = [\Delta],$$

we have from equation (1)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v1 \\ v2 \\ v3 \\ v4 \\ v5 \\ v6 \end{bmatrix} = \begin{bmatrix} \Delta 1 \\ \Delta 2 \end{bmatrix}$$

and the weight matrix is:

$$W = diag\left[{}^1\!/_{\sigma_{11}^2}, {}^1\!/_{\sigma_{22}^2}, {}^1\!/_{\sigma_{33}^2}, {}^1\!/_{\sigma_{44}^2}, {}^1\!/_{\sigma_{55}^2}, {}^1\!/_{\sigma_{66}^2} \right]$$

$$W^{-1} = \left[\sigma_{11}^2, \sigma_{22}^2, \sigma_{33}^2, \sigma_{44}^2, \sigma_{55}^2, \sigma_{66}^2 \right]$$

$$W^{-1} = \begin{bmatrix} 1.43 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.43 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.18 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.43 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.71 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.43 \end{bmatrix}$$

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TABLE 2: ANGULAR REDUCTION AND DETERMINATION OF STANDARD ERROR (σ)

ANGLE (θ)	L	R	Mean (θ_m)	$\theta_m - \theta_i$			Std. error (σ^2)
				V= $\theta_m - \theta_l$	V= $\theta_m - \theta_r$	$\left(\frac{\theta_m - \theta_r}{\theta_m - \theta_l}\right)/2$	
1	19° 11' 23"	19° 11' 22"	19° 11' 22.5"	0.50’’	0.50’’	0.5’’	0.7
6	29° 58' 13"	29° 58' 14"	29° 58' 13.5"	0.50’’	0.50’’	0.5’’	0.7
2	117° 37' 1"	117° 37' 2"	117° 37' 1.5"	0.50’’	0.50’’	0.5’’	0.7
3	43° 11' 22"	43° 11' 30"	43° 11' 26.0"	4.00’’	4.00’’	4.00’’	5.7
4	48° 21' 31"	48° 21' 30"	48° 21' 30.5"	0.50’’	0.50’’	0.5’’	0.7
5	101° 40' 8"	101° 40' 6"	101° 40' 7.0"	1.00’’	1.00’’	1.0’’	1.4

Where $\sigma^2 = \left[\left(\sqrt{v_m^2 - v_i^2}\right)/r_n - 1\right] = \left[\left(\sqrt{\left(\frac{\theta_m - \theta_r}{\theta_m - \theta_l}\right)/2}\right)/r_n - 1\right], r_n = 2$

2. *The solution to the normal equations is given as:*

$$M = \left[B \times W^{-1} \times B^T\right]$$

$$M = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1.43 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.43 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.18 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.43 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.71 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.43 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 3.04 & 0 \\ 0 & 3.57 \end{bmatrix}$$

3. *The Lagrange*

The vector of misclosure is given as:

$$\Delta = \begin{bmatrix} 10 \\ 9 \end{bmatrix}$$

$$K = \left[M^{-1} \times \Delta\right]$$

$$K = \begin{bmatrix} 3.04 & 0 \\ 0 & 3.57 \end{bmatrix} \begin{bmatrix} 10 \\ 9 \end{bmatrix}$$

$$K = \begin{bmatrix} 3.2895'' \\ 2.5210'' \end{bmatrix}$$

4. *The residuals are given as:*

$$V = W^{-1} B^T K$$

$$V = \begin{bmatrix} 4.7039" \\ 4.7039" \\ 0.5921" \\ 3.6050" \\ 1.7899" \\ 3.6050" \end{bmatrix}$$

Variance

$$\sigma^2 = \left[\frac{V^T V}{r}\right]$$

$$\sigma^2 = [18.4503]$$

5. *The adjusted observations*

Finally, adding the residuals to the observations, we obtain the adjusted observations: $\bar{l}$=L+V

TABLE 4: ADJUSTED OBSERVATIONS

ANGLE (θ)	OBSERVED ANGLE (θ)	Residuals (v)	CORRECTED ANGLE (θ)
1	19° 11' 22.5"	4.7039"	19° 11' 27.20"
2	117° 37' 1.5"	4.7039"	117° 37' 6.20"
3	43° 11' 26.0"	0.5921"	43° 11' 26.59"
4	48° 21' 30.5"	3.6050"	48° 21' 34.10"
5	101° 40' 7.0"	1.7899"	101° 40' 8.8"
6	29° 58' 13.5"	3.6050"	29° 58' 17.11"
Σ	359° 59' 41.0"	19.00"	360° 0' 0.00"

6. *Azimuth of lines:*

Bearing of line,

β= tan^{-1} (X2 - X1)/(Y2 - Y1)

1. β(AB)=tan^{-1}(507,450.410-507,327.641)/(92,079.322--91,756.439)=20° 49' 5.58"

ANGLE (θ2) = 117° 37' 6.2"

2. β(BC)=β(BA)–θ2=20°49'5.58"+180-117°37'6.2"= =83° 11' 59.38”

ANGLE (θ3+θ4)=43°11'26.59"+48°21'34.10"=91° 33' 0.69"

3. β(CD)=β(CB)-(θ3+θ4)=83°11'59.38"+180-91°33' 7.71" = 171° 38' 58.69"

4. β(AC)=β(AB)+θ1=20° 49' 5.58" + 19° 11' 27.20"= =40° 00' 32.78"

ANGLE (θ5) = 101° 40' 8.80".

5. β(AD)=β(BC)+θ6=(40°00'32.78"+29°58'17.11’’)= = 69° 58' 49.89".

7. *Coordinates of points*

Distance between two points (A and B) equals to

$$= \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$$

$$= \sqrt{(507,450.410 - 507,327.641)^2 + (92,079.322 - 91,756.439)^2}$$

$AB = 345.4354m$

$$\frac{AB}{\mathrm{Sin}(\theta_3)} = \frac{BC}{\mathrm{Sin}(\theta_1)}$$

$$BC = (345.435X \;\; \mathrm{Sin}19^\circ 11' 27.2'') / \mathrm{Sin}\,43^\circ 11' 26.59'' = 165.905m$$

$$\frac{BC}{\mathrm{Sin}(\theta_3)} = \frac{AC}{\mathrm{Sin}(\theta_2)}$$

$$AC = (345.435X \;\; \mathrm{Sin}117^\circ 37' 6.2'') / \mathrm{Sin}\,43^\circ 11' 26.59'' = 447.197m$$

$$\frac{AD}{\mathrm{Sin}(\theta_4)} = \frac{AC}{\mathrm{Sin}(\theta_5)}$$

$$AD = (447.197X \;\; \mathrm{Sin}\,48^\circ 21' 34.2'' 10 / \mathrm{Sin}101^\circ 40' 8.80'' = 341.256m$$

$$\frac{CD}{\mathrm{Sin}(\theta_6)} = \frac{AD}{\mathrm{Sin}(\theta_4)}$$

$$CD = (341.256X \;\; \mathrm{Sin}\,29^\circ 58' 17.11'') / \mathrm{Sin}\,48^\circ 21' 34.1'' = 228.120m$$

Easting of C = (Easting of A) + (AC x Sinβ(AC)) = 507,615.148mE.
Northing of C = (Northing of A) + (AC x Cosβ(AC)) = 92,098.966mN.
Easting of C = (Easting of B) + (BC x Sinβ(BC)) = 507,615.147mE.
Northing of C = (Northing of B) + (BC x Cosβ(BC)) = 92,098.966mN.
Easting of D = (Easting of A) + (AD x Sinβ(AD)) = 507,648.277mE.
Northing of D = (Northing of A) + (AD x Cosβ(AD)) = 91,873.264mN.
Easting of D = (Easting of C) + (CD x Sinβ(CD)) = 507,648.260mE.
Northing of D = (Northing of C) + (CD x Cosβ(CD)) = 91,873.270mN.

B. Adjustment of a double braced quadrilateral using least square condition equation

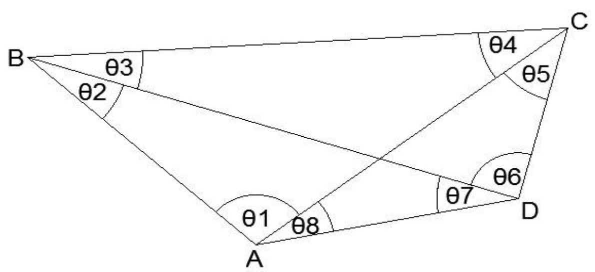

Fig. 2: Double Braced Quadrilateral

The measurements for a double braced quadrilateral ABCD are given below. Using this data, calculate the coordinates of station C and D.

TABLE 5: FIELD OBSERVATION DATA

ANGLE (θ)	STN FROM	FACE	Degree	mins	sec	STN TO
8	A	L	0	00	00	D
		L	29	58	15	C
		R	209	58	16	C
		R	180	00	05	D
1	A	L	10	00	00	C
		L	29	11	26	B
		R	209	11	25	B
		R	190	0	05	C
2	B	L	20	00	00	D
		L	84	39	23	A
		R	264	39	24	A
		R	200	00	05	D
3	B	L	0	00	00	D
		L	52	57	42	C
		R	232	57	43	C
		R	180	00	05	D
5	C	L	0	00	00	A
		L	48	21	33	B
		R	228	21	34	B
		R	180	00	05	A
4	C	L	10	00	00	A
		L	53	11	31	D
		R	233	11	32	D
		R	190	00	10	A
7	D	L	0	00	00	B
		L	66	10	57	A
		R	246	10	58	A
		R	180	00	04	B

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III. Angular Reduction And Calculation Of Standard Error (σ)

TABLE 6: ANGULAR REDUCTION AND CALCULATION OF STANDARD ERROR (Σ)

ANGLE (θ)	θ		θ_m	$\theta_m - \theta_i$			S. E. (σ)
	L	R		V= $\theta_m - \theta_l$	V= $\theta_m - \theta_r$	$\left(\dfrac{\theta_m - \theta_r}{\theta_m - \theta_l}\right)/2$	
8	29° 58' 15"	29° 58' 11"	29° 58' 13.0"	2.00	2.00	2.0’’	2.8
1	19° 11' 26"	19° 11' 20"	19° 11' 23.0"	3.00	3.00	3.00’’	4.2
2	64° 39' 23"	64° 39' 19"	64° 39' 21.0"	2.00	2.00	2.00’’	2.8
3	52° 57' 42"	52° 57' 38"	52° 57' 40.0"	2.00	2.00	2.00’’	2.8
5	48° 21' 33"	48° 21' 29"	48° 21' 31.0"	2.00	2.00	2.00’’	2.8
4	43° 11' 31"	43° 11' 22"	43° 11' 26.5"	4.50	4.50	4.50’’	6.4
7	66° 10' 57"	66° 10' 54"	66° 10' 55.5"	1.50	1.50	1.50’’	2.1
6	35° 29' 15"	35° 29' 10"	35° 29' 12.5"	2.50	2.50	2.50’’	3.5

TABLE 7: OBSERVED ANGLES AND STANDARD ERRORS

ANGLE (θ)	OBS. VALUE (θ)	STD ERROR (σ^2)	STNn	E	N
1	19° 11' 23.0"	4.2	A	507327.643	91756.449
2	64° 39' 21.0"	2.8	B	507450.402	92079.315
3	52° 57' 40.0"	2.8			
4	43° 11' 26.5"	6.4			
5	48° 21' 31.0	2.8			
6	35° 29' 12.5"	3.5			
7	66° 10' 55.5"	2.1			
8	35° 29' 12.5"	2.8			

The number of total observations (n) = 8 Angles.

The number of necessary observations (no) = 2 (No of new points) = 2(2) = 4.

Hence, the number of conditions (r) = n –no = 4.

1. Condition equations

There are four condition equation:

1) Triangular condition equations are:

Fig. 2: ABCD

* Opposites angles (θ 7 +α 8) = (θ 3 + θ 4)

Also (θ 7 + v7 + θ 8 + v8) = (θ 3 + v3 + θ 4 + v4)

(θ 7+θ 8)-(θ 3+θ 4)=(v3+v4)-(v7+v8)=Δ1

where, Δ1 = (θ 7+θ 8)-(θ 3+θ 4)=(96 9' 8.5"-96 9' 6.5") = =2.0"

* Opposites angles (θ 5 + θ 6) = (θ 1 + θ 2)

Also (θ5 + v5 + θ6 + v6) = (θ1 + v1 + θ2 + v2)

(θ5 +θ6) - (θ1 + θ2) = (v1 + v2) - (v5 + v6) = Δ2

where, Δ2 = (θ5 + θ6) - (θ1 + θ2) = Δ2 = (83 50' 43.5" - 83 50' 44.0") = -0.5".

Sum of angles in Fig. 2 ABCD:

v1+v2+v3+v4+v5+v6+v7+v8=Δ3,

where Δ3 =360°-(θ1+θ2+θ3+θ4+θ5+θ6+θ7+θ8) Δ3 = 17.5"

2) Side condition equations are:

Taking intersection of two diagonals as a pole for the figure ABCD:

δ1v1–δ2v2+δ3v3–δ4v4+δ5v5–δ6v6+δ7v7–δ8v8=Δ4

where:

Δ4=Log sin (θ1)+Log sin(θ3)+Log sin(θ5)+Log sin(θ7)-Log sin(θ2)–Log sin(θ4)–Log sin(θ6)–Log sin(θ8)=-1.213E-6

δi is the difference in log sin (θi) for a one second arc,

δi = [Log Sin (θ+1") - Log Sin (θ)]x 10-6

TABLE 8: LOG SIN OF OBSERVATIONS

ANGLE (θ)	OBS. VALUE (θ)	Log Sin (θ+1)	Log Sin (θ)	δi (x 10^{-6})
1	19° 11' 23.0"	-0.483197895	-0.483203945	6.050
2	64° 39' 21.0"	-0.043949323	-0.04395032	0.997
3	52° 57' 40.0"	-0.097872079	-0.097873668	1.589
4	43° 11' 26.5"	-0.164669543	-0.164671786	2.243
5	48° 21' 31.0"	-0.126492515	-0.126494387	1.872
6	35° 29' 12.5"	-0.236183256	-0.236186209	2.953
7	66° 10' 55.5"	-0.038657043	-0.038657972	0.929
8	29° 58' 13.0"	-0.301416793	-0.301420444	3.651

$$\begin{bmatrix} 1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 6.05\times10^{-6} & 0.997\times10^{-6} & 1.589\times10^{-6} & 2.243.05\times10^{-6} & 1.872\times10^{-6} & 2.953\times10^{-6} & 0.929\times10^{-6} & 3.651\times10^{-6} \end{bmatrix}$$

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$$\Delta = \begin{bmatrix} 2 \\ -0.5 \\ 17.5 \\ -1.213\times10^{-6} \end{bmatrix}$$

which is in the general form Bv=Δ and the weight matrix is:

W= diag. [$1/\sigma^2_1$ $1/\sigma^2_2$ $1/\sigma^2_3$ $1/\sigma^2_4$ $1/\sigma^2_5$ $1/\sigma^2_6$ $1/\sigma^2_7$ $1/\sigma^2_8$]

W-1 = diag. [σ21 σ22 σ23 σ24 σ25 σ26 σ27 σ28]

$$W^{-1} = \begin{bmatrix} 17.64 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7.84 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7.84 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 40.96 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7.84 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 12.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4.41 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7.84 \end{bmatrix}$$

2. *The normal equations is given as:*

$$M = BW^{-1}B^T$$

$$M = \begin{bmatrix} 74.2800 & 0 & -23.3200 & 0.0000 \\ 0 & 32.3400 & 0.9800 & 0.0000 \\ -23.3200 & 0.9800 & 106.6200 & 0.0003 \\ 0.0000 & 0.0000 & 0.0003 & 0.0000 \end{bmatrix}$$

3. *The Lagrange multipliers are:*

$$K = M^{-1}\Delta$$

$$K = \begin{bmatrix} 74.2800 & 0 & -23.3200 & 0.0000 \\ 0 & 32.3400 & 0.9800 & 0.0000 \\ -23.3200 & 0.9800 & 106.6200 & 0.0003 \\ 0.0000 & 0.0000 & 0.0003 & 0.0000 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ -0.5 \\ 17.5 \\ -1.213\times10^{-6} \end{bmatrix}$$

$$K = \begin{bmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ -4.6851 \end{bmatrix}\times10^{5}$$

$$V = W^{-1}B^T K$$

$$V = \begin{bmatrix} -8.0783'' \\ 7.5157'' \\ 5.3412'' \\ -5.2204'' \\ 0.3635'' \\ 6.0111'' \\ 6.3458'' \\ 5.2214'' \end{bmatrix}$$

4. *The adjusted observations*

Finally, adding the residuals to the given observations, we obtain the adjusted observations: L=L+v.

TABLE 9: FINAL ADJUSTED OBSERVATIONS

ANGLE (θ)	OBSERVED ANGLE (θ)	Residuals (v)	CORRECTED ANGLE (θ)
1	19° 11' 23.0"	-8.0783"	19° 11' 14.9217"
2	64° 39' 21.0"	7.5157"	64° 39' 28.5157"
3	52° 57' 40.0"	5.3412"	52° 57' 45.3412"
4	43° 11' 26.5"	-5.2204"	43° 11' 21.276"
5	48° 21' 31.0"	0.3635"	48° 21' 31.3635"
6	35° 29' 12.5"	6.0111"	35° 29' 18.5111"
7	66° 10' 55.5"	6.3458"	66° 11' 01.8458"
8	29° 58' 13.0"	5.2214"	29° 58' 18.2214"
Σ	359 59' 42.5"	17.5"	360° 00' 0.00"

5. *Azimuth of lines:*

Bearing of line, β = tan-1 (X2 - X1)/(Y2 - Y1)

β(AB)=tan⁻¹ (507,450.402 - 507,327.643)/(92,079.315 - 91,756.443) = 20° 49' 5.58"

ANGLE (θ2+θ3) = 64° 39' 28.52" + 52° 57' 45.34" = 117° 37' 13.84"

β(BC) = 20° 49' 5.58" + 180 - 117° 37' 13.84" = 83° 11' 51.74"

ANGLE (θ4+θ5) = 43° 11' 21.28" + 48° 21' 31.36" = 91° 32' 52.64" β(CD) = 83° 11' 51.74" + 180 - 91° 32' 52.64" = 171° 38' 59.10"

ANGLE (θ6+θ7) = 35° 29' 18.51" + 66° 11' 01.85" = 101° 40' 20.36" β(DA) = 171° 38' 59.10" + 180 - 101° 40' 20.36" = 249° 58' 38.74"

ANGLE (θ8+θ1) = 29° 58' 18.22" + 19° 11' 14.92" = 49° 9' 33.14" β(AD) = 20° 49' 5.58+ 49° 9' 33.14" = 69° 58' 38.72"

ANGLE (θ1) = 19° 11' 14.92"

β(AC) = 20° 49' 5.58" + 19° 11' 14.92" = 40° 00' 20.50"

β(BD) = 20° 49' 5.58" + 180 - 64° 39' 28.52"= 136° 09' 37.06"

6. *Coordinates of points*

$$AB = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$$

$$AB = \sqrt{(507{,}450.410 - 507{,}327.641)^2 + (92{,}079.322 - 91{,}756.439)^2}$$

$$AB = 345.4354m$$

$$\frac{BC}{\mathrm{Sin}(\theta_1)} = \frac{AB}{\mathrm{Sin}(\theta_4)} \qquad AC = AB\,\mathrm{Sin}(\theta_1)/\mathrm{Sin}(\theta_4)$$

$$BC = (345.435X \ \ \mathrm{Sin}\,19°11'14.92)/\mathrm{Sin}\,43°11'21.28''$$

$$BC = 165.881m$$

$$\frac{AC}{\mathrm{Sin}(\theta_2+\theta_3)}=\frac{AB}{\mathrm{Sin}(\theta_4)}$$

$$AC = AB\,\mathrm{Sin}(\theta_2+\theta_3)/\mathrm{Sin}(\theta_4)$$

$$AC = \left(345.435\,X\ \ \mathrm{Sin}\,117°37'13.86"\right)/\mathrm{Sin}\,43°11'21.28"$$
$$AC = 447.200m$$

$$\frac{BD}{\mathrm{Sin}(\theta_1+\theta_8)}=\frac{AB}{\mathrm{Sin}(\theta_7)}$$

$$BD = AB\,\mathrm{Sin}(\theta_1+\theta_8)/\mathrm{Sin}(\theta_7)$$

$$BD = \left(345.435\,X\ \ \mathrm{Sin}\,49°09'33.14"\right)/\mathrm{Sin}\,66°11'1.85"$$

$$BD = 285.657m$$

$$\frac{AC}{\mathrm{Sin}(\theta_7+\theta_6)}=\frac{AD}{\mathrm{Sin}(\theta_5)}$$

$$AD = AC\,\mathrm{Sin}(\theta_5)/\mathrm{Sin}(\theta_7+\theta_6)$$

$$AD = \left(447.20\,X\ \ \mathrm{Sin}\,48°21'31.36"\right)/\mathrm{Sin}\,101°40'20.37"$$

$$AD = 341.258m$$

$$Easting\ of\ C = (Easting\ of\ \mathrm{A}) + (AC\ x\ Sin\beta(AC)) = 507{,}615.123mE$$
$$Northing\ of\ C = (Easting\ of\ \mathrm{A}) + (AC\ x\ Cos\beta(AC)) = 92{,}098.985mN$$
$$Easting\ of\ D = (Easting\ of\ \mathrm{A}) + (\mathrm{AD}\ x\ Sin\beta(AD)) = 507{,}648.273mE$$
$$Northing\ of\ D = (\mathrm{Northing}\ of\ \mathrm{A}) + (CD\ x\ Cos\beta(CD)) = 91{,}873.282mN$$
$$Easting\ of\ C = (Easting\ of\ B) + (BC\ x\ Sin\beta(BC)) = 507{,}615.123mE$$
$$Northing\ of\ C = (\mathrm{Northing}\ of\ B) + (BC\ x\ Cos\beta(BC)) = 92{,}098.970mN$$
$$Easting\ of\ D = (Easting\ of\ \mathrm{B}) + (\mathrm{AD}\ x\ Sin\beta(AD)) = 507{,}648.268mE$$
$$Northing\ of\ D = (\mathrm{Northing}\ of\ \mathrm{B}) + (CD\ x\ Cos\beta(CD)) = 91{,}873.283mN$$

8. Conclusion

Comparisons of the solutions derived from Observation 1 and Observation 2 showed a high degree of agreement. Therefore, since the solutions from the adjustment of the single and double brace quadrilateral are the same we conclude that the adjustment is a reliable. In summary, some of the primary conditions for LS adjustment among others are that:

(i) the number of field observations must exceed the number of parameters to be determined;

(ii) the number of observation equations formed must be equal to the number of field observations;

(iii) the number of condition equations formed must equal the difference between the number of observations and the number of unknown parameters to be determined.

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